and

$$(C_L)_{\rm arc} = 8\epsilon_2 \frac{\alpha}{c'^2}[(kcc' - \frac{c'^2}{4})\sin^{-1}(\frac{c}{c'})^{1/2} + \frac{kcc'}{2}\sin(2\sin^{-1}(\frac{c}{c'})^{1/2}) + \frac{c'^2}{16}\sin(4\sin^{-1}(\frac{c}{c'})^{1/2})]/(1 + 4k(\frac{c}{c'})^2) + \frac{2gz}{V^2} \quad (7)$$

where ϵ_1 and ϵ_2 are efficiency factors which are liable to depend on aspect ratio, foil profile, Reynold's number, etc.

In Eq. (7) c', which is a measure of cavity length, is still unknown. Some work has been done on producing expressions for the cavity length. Thomsen[1] produced an expression from potential flow conditions but this result is inadequate with respect to both velocity and angle dependence. Swales et al.[2] evaluated the cavity length from centrifugal considerations for a circular cylinder and suggested that an extension to aerofoils was possible. This is not easily accomplished but the concept does have one major advantage, namely, that the proposals for shape mean that the free surface boundary condition on the cavity is adequately satisfied. Following the concept leads to the conclusion that c' at the half span point, $z = -c$, may be expressed by

$$c' = c + \frac{a\alpha V^2}{gc} - bV(\frac{2\alpha}{g})^{1/2} \quad (8)$$

where a and b are constants and g is the acceleration due to gravity. The constants a and b may be determined from measurement of cavity lengths in a water channel and the function k from pressure measuremerts. The efficiency factor ϵ_1 is chosen to give the nonventilated lift slope and ϵ_2 was initially given the value unity since the ventilation greatly reduces the three-dimensional nature of the flow on the wetted face. The value of ϵ_2 was modified in the final runs to take into account changes in effective submergence of the foils. Using these values the lift distributions are shown in Fig. 1. The curves shown in this are compared with the forces measured in the Leeds recirculating water channel[3-5] for decreasing angles of incidence after ventilation; the discontinuity is the jump in side force which occurs on washout.

The good agreement between analysis and experiment refutes the suggestion that the force change caused by ventilation is solely the effect of the loss of the suction contribution on the low pressure face and confirms the hypothesis that the cavity influences the fluid flow to such an extent that the high pressure face in preventilated flow can become the suction face after ventilation. This point was confirmed by the experimental pressure measurements. The theory also explains the force reversal which can occur on ventilation since at inception the force drops to a point on the curves plotted in Fig. 1, where the force coefficient may be negative. The circumstances for which reversal actually occurs are correctly reproduced.

References

[1]Thomsen, P., "Cavity Shape and Drag in Ventilated Flow; Theory and Experiment," TRG-156-SR-2, Feb. 1963, Office of Naval Research, Washington, D.C.

[2]Swales, P. D., Cole, B. N., and Smith, G. L., "Some Aspects of the Ventilation of Surface Piercing Struts and Foils," *Transactions of the RINA*, Vol. 113, 1971, pp. 365–374.

[3]Hunter, B., Swales, P. D., and Cole, B. N., "Variable Pressure Water Channel," *The Engineer,* Vol. 225, March 1968, pp. 515–519.

[4]Swales, P. D., Wright, A. J., McGregor, R. C., and Cole, B. N., "Ventilation Studies on Two Biogival Struts," Rept. TP/DREA/3, Nov. 1971, Dept. of Mechanical Engineering, Leeds Univ., Leeds, England.

[5]Wright, A. J. and McGregor, R. C., "Ventilation Studies on NACA 0012 and NACA 16-021 Struts," Rept. TP/DREA/4, March 1972, Dept. of Mechanical Engineering, Leeds Univ., Leeds, England.

Axisymmetrical Turbulent Boundary Layer along a Slender Cylinder

Chang-yu Liu* and Yau-tien Dai†

Ship Hydrodynamic Laboratory, National Taiwan University, Taipei, Taiwan

Introduction

THICK axisymmetrical turbulent boundary layers along slender cylinders are of importance for the estimation of the skin-friction drag of ships and rocket bodies. Investigations were carried out by Rao,[1] White,[2] and Cebici,[3] on the behavior of mean velocity profile showed that it is quite different from that of flat plate. However, these profiles also exhibit a logarithmic nature near the surface. Recently, Joseph et al.[4] showed that the mean velocity profile can also be fitted by power law, which depends on the radius of the cylinder.

Employing Joseph's criterion, we can derive a method to predict the parameters of a thick axisymmetrical turbulent boundary layers. Good comparisons are obtained with existing experimental data.

Analysis

The steady momentum integral equation of incompressible fluid flow over a circular cylinder, with its axis parallel to the stream, can be written as

$$\frac{d}{dx}\int_0^\delta \left(1 + \frac{y}{a}\right) \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = 1/2\, C_f \qquad (1)$$

with the boundary conditions

$$u = 0, \text{ at } r = a$$
$$u = U \text{ as } r \to \infty$$

where x is the coordinate measured along the axis of the cylinder, y is the coordinate normal to the surface, u is the mean velocity component in the x direction, U is the freestream velocity, a is the radius of the cylinder, C_f is the local skin-friction coefficient, and δ is the boundary-layer thickness measured from the surface of the cylinder. An experimental study by Joseph et al.[4] showed that the velocity profiles can be adequately described by the power law relation

$$u/U = (y/\delta)^{1/n} \qquad (2)$$

where n can be expressed by the form

$$(n - 7)\,a^{1.26} = 0.62 \text{ for } a < 0.75 \text{ in.}$$
$$n = 7 \text{ for } a \geq 0.75 \text{ in.} \qquad (3)$$

The aforementioned relation is valid over a certain range of Reynolds number. Substituting Eq. (2) into Eq. (1) and

Received September 19, 1972.

Index categories: Airplane and Component Aerodynamics; Boundary Layers and Convective Heat Transfer—Turbulent.

* Professor.

† Associate Professor.

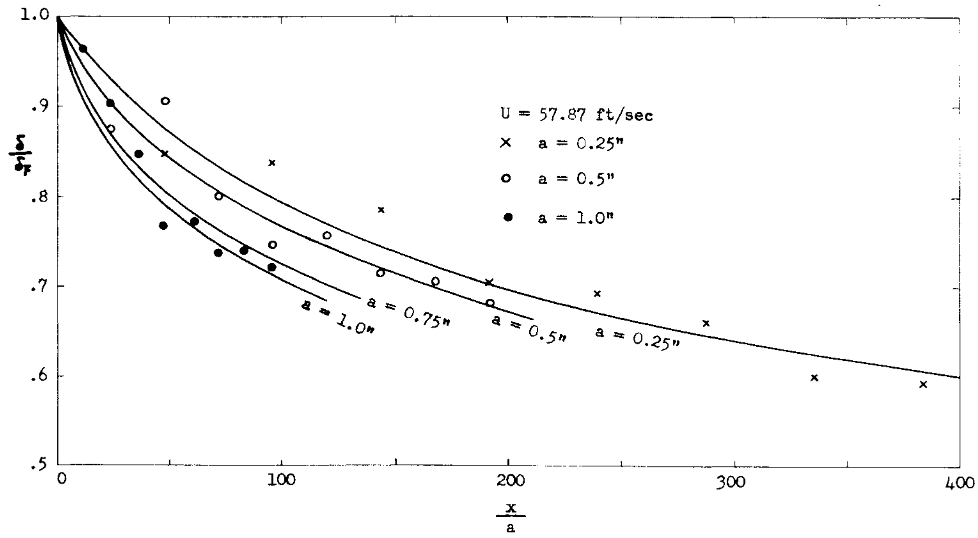

Fig. 1 Development of boundary-layer thickness (U = 57.87 fps).

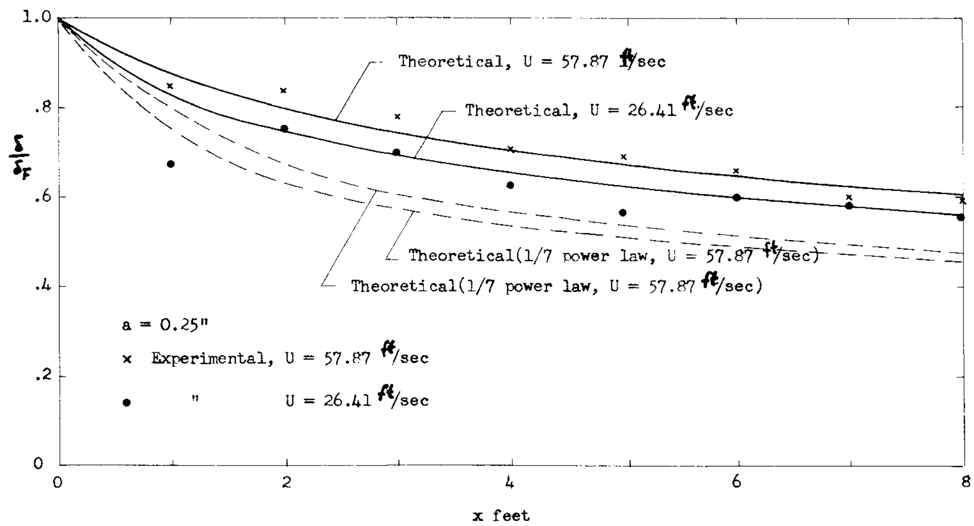

Fig. 2 Development of boundary-layer thickness (a = 0.25 in.).

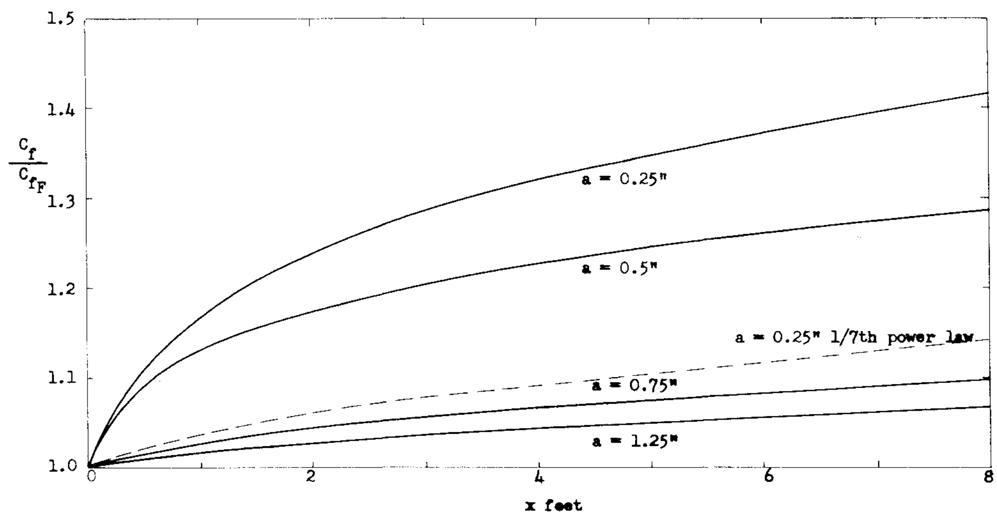

Fig. 3 Variation of local skin-friction coefficient along the surface of the cylinder (U = 57.87 fps).

integrating with respect to y/a from $y = 0$ to δ, we have

$$\frac{n}{(n+1)(n+2)}\frac{d\delta}{dx} + \frac{n}{a(2n+1)(2n+2)}\frac{d\delta^2}{dx} = \frac{1}{2}C_f \quad (4)$$

Equation (4) can not be integrated again without the knowledge of C_f in terms of δ. Assume the relation

$$u/u_\tau = C(n)(yu_\tau/\nu)^{1/n} \quad (5)$$

which has been proved to be a good representation of the mean velocity profiles of pipe flow, it is also valid for the present case, where u_t is the friction velocity, $C(n)$ is function of n only, its value having been tabulated by Schlichting[5] book, and ν is kinematic viscosity of the fluid. Employing the boundary condition at $y = \delta$, we can transform Eq. (5) into the form

$$C_f = 2C^{-2n/(n+1)}(\delta U/\nu)^{-2/(n+1)} \tag{6}$$

which gives a relation of C_f in terms of δ. After substituting Eq. (6) into Eq. (4) and integrating, we have

$$\left[\frac{n}{(n+2)(n+3)} + \frac{n}{2(2n+1)(n+2)}\frac{R_\delta}{R_a}\right] \times R_\delta^{(n+3)/(n+1)} = D(n)R_x \tag{7}$$

where

$$R_\delta = \frac{\delta U}{\nu}, \quad R_a = \frac{aU}{\nu}, \quad R_x = \frac{xU}{\nu}, \quad D(n) = C^{\frac{-2n}{n+1}}$$

For the case of flat plate $R_a = \infty$, Eq. (7) can be reduced to

$$R_{\delta_F}^{1.25} = 0.288R_x, \quad (n = 7) \tag{8}$$

where the subscript, F, denotes the quantity for the case of flat plate. Dividing Eq. (7) by Eq. (8), we have

$$\left[\frac{n}{(n+2)(n+3)} + \frac{n}{2(2n+1)(n+2)}\frac{R_\delta}{R_a}\right] \times \frac{R_\delta^{(n+3)/(n+1)}}{R_{\delta_F}^{1.25}} = \frac{D(n)}{0.288} \tag{9}$$

Equation (9) represents a comparison between δ and δ_F at the same value of R_x. Assuming $n = 7$, we can reduce Eq. (9) into a simpler form

$$\delta/\delta_F = (1 + \delta/3a)^{-0.8} \tag{10}$$

which was obtained by Landweber.[6] It was shown by Joseph et al.[4] that n is no longer equal to 7 for cylinder radii $a < 0.75$ in. Equation (10) will not give accurate results for small cylinders. This part will be discussed later. For a given radius of a circular cylinder, n can be calculated by Eq. (3). Knowing the value of R_x, we can calculate $R_{\delta F}$ by Eq. (8), than the corresponding R_δ can be calculated by Eq. (9). Figure 1 is a comparison of the boundary-layer thickness calculated by the present method to the experimental data of Joseph et al.[4] Good results are obtained. Figure 2 shows the results for $a = 0.25$ in. Landweber's[6] and the present theoretical results are also shown. For this case $n = 10.1$. The present method shows better agreement with the experimental data than do Landweber's[6] predictions. Other boundary-layer parameters, such as displacement thickness and momentum thickness, can also be predicted by the power law relation.

Knowing the development of δ, one can easily calculate the local skin-friction coefficient by Eq. (6); results are shown in Fig. 3. For the case $a = 0.25$ in., $n = 7$, the calculated results are also shown. It is seen that there is about 20% difference between the two. Unfortunately, no experimental data were available to the authors, however, the tendency is seen to be reasonable.

References

[1] Rao, G. N. V., "The Law of Wall in a Thick Axisymmetric Turbulent Boundary Layer," *Transactions of the ASME: Journal of Applied Mechanics*, Vol. 34, 1967, pp. 237-238.

[2] White, F. M., "The Axisymmetric Turbulent Boundary Layer on an Extremely Long Cylinder," Rept. 1058, Nov. 1969, Navy Under Water Sound Lab., Washington, D.C.

[3] Cebeci, T., "Laminar and Turbulent Boundary Layers on Slender Bodies of Revolution in Axial Flow," *Transactions of the ASME: Journal of Basic Engineering*, 1970, Vol. 92, No. 3, pp. 545-554.

[4] Joseph M. C., McCorquodale, J. A., and Iriclhar, K. S., "Power Law for Turbulent Cylindrical Boundary Layers," *The Aeronautical Journal*, Vol. 75, Jan. 1971, pp. 46-48.

[5] Schlichting, H., *Boundary Layer Theory*, 6th ed., McGraw-Hill, New York, 1968, p. 75.

[6] Landweber, L., "Effect of Transverse Curvature on Frictional Resistance," Rept. 689, 1949, David W. Taylor Model Basin, Washington, D.C.

Asymptotic Suction Flow of Power-Law Fluids

Chang-yu Liu*

National Taiwan University, Taipei, Taiwan, Republic of China

Introduction

SUCTION has been approved to be a powerful method for boundary-layer control for the purpose of increasing lift and reducing drag. Many theoretical as well as experimental studies have been reported. A surprisingly simple solution can be obtained when the velocity components are independent of the longitudinal coordinate. Among these solutions, Schlichting obtained a solution for the flow over a flat plate at zero incidence with uniform suction. Liu[2] obtained an unsteady asymptotic suction solution when the external flow is an exponential function of time. This Note presents a class of asymptotic suction solution for the flow of power-law fluids over a flat plate.

Basic Equations and Solutions

Under the assumptions of study and two-dimensional asymptotic suction flow, the momentum equation for the flow of power-law fluids over a flat plate can be expressed as

$$V_0\frac{du}{dy} = \frac{d}{dy}\left(k\left|\frac{du}{dy}\right|^{N-1}\frac{du}{dy}\right) \tag{1}$$

with the boundary conditions

$$u = 0 \quad \text{at} \quad y = 0$$
$$u = U \quad \text{as} \quad y \to \infty$$

where y is the coordinate normal to the plate, V_0 and u are, respectively, the velocity components normal and along the plate. N and K are parameters related to power-law model.

Equation (1) can be integrated with respect to y, yielding

$$V_0(U - u) = -K\left|\frac{du}{dy}\right|^N \tag{2}$$

in which the constant of integration is determined by the condition that at $y \to \infty$, $u \to U$ and $du/dy = 0$. This can be integrated again by separation of variables, as evident

Received August 31, 1972; revision received February 7, 1973.

Index category: Boundary Layers and Convective Heat Transfer—Laminar.

*Professor, Ship Hydrodynamic Laboratory.